operating conditions where these purely gasdynamic considerations could explain experimental findings.

Finally, the authors wish to raise the question of stability of solutions which exhibit two choking points separated by a shock. A situation of this sort may well be found unstable, in which case the present study may be relevant to fluctuations observed in some thrusters.

Reference

[1] Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vol. 1, Ronald Press, New York, 1953, p. 230.

Recursive Filter Initialization

RICHARD H. BATTIN,* STEVEN R. CROOPNICK,† AND JAMES M. HABBE‡
Charles Stark Draper Laboratory, Cambridge, Mass.

THE application of recursive filtering techniques to space navigation requires that the associated state vector and error covariance matrix be specified both initially and at those times when reinitialization of the filter is required to cope with problems of divergence and instability. A fairly common initialization method, and one that is used in Apollo navigation,[1] is to postulate that the initial errors in the state are uncorrelated and spherically distributed, which implies that the initial covariance matrix of estimation errors is diagonal. The true state vector errors are, of course, correlated and several measurements must be processed before the covariance matrix begins to exhibit proper cross-correlations.

The purpose of this Note is to describe an initialization technique which partially accounts for these correlations, considerably reduces the undesirable transient effects of the first few measurements, and acts to inhibit filter divergence when the interval between measurements is inordinately large. The improvement in navigation performance is quite dramatically illustrated (in the second of the two examples considered below) for a space shuttle vehicle navigating with the aid of somewhat sparsely distributed ground beacons.

The method consists of including in the initial covariance matrix the effect of a number of pseudo-measurements of certain orbital parameters. Specifically, define the measurement geometry vector $\mathbf{b}$ to represent, to a first order of approximation, the variation in the measured quantity Q which would result from variations in the components of the state vector $\mathbf{x}$. That is

$$\mathbf{b} = (\partial Q/\partial \mathbf{x})^T \tag{1}$$

By identifying Q with an orbital parameter expressed in terms of the components of the state vector and calculating the necessary partial derivatives, the measurement geometry vector $\mathbf{b}$ may be determined from the initial estimate of the state. Although no direct measurement of the parameter is made, this measurement information can be adjoined to the error covariance matrix by assuming a reasonable value for a measurement variance and updating the covariance matrix as though an actual measurement had been performed.

Two orbital parameters that have proved to be particularly useful for this purpose are the total energy

$$E = v^2/2 - \mu/r \tag{2}$$

and the angular momentum

$$h = |\mathbf{h}| = |\mathbf{r} \times \mathbf{v}| \tag{3}$$

Assuming the state vector to be six-dimensional

$$\mathbf{x}^T = (\mathbf{r}^T, \mathbf{v}^T) \tag{4}$$

the corresponding geometry vectors $\mathbf{b}_E$ and $\mathbf{b}_h$ are readily obtained from Eq. (1) as

$$\mathbf{b}_E = \begin{pmatrix} \mu \mathbf{r}/r^3 \\ \mathbf{v} \end{pmatrix} \tag{5}$$

$$\mathbf{b}_h = (1/h)\begin{pmatrix} \mathbf{v} \times \mathbf{h} \\ \mathbf{h} \times \mathbf{r} \end{pmatrix} \tag{6}$$

This technique was applied[2] to the problem of navigating a spacecraft during the midcourse phase of a moon-to-Earth flight. A desired or nominal trajectory for the trans-Earth path was assumed known and the vehicle given the necessary position and velocity to achieve this trajectory with the exception of small initial position and velocity perturbations at the time of the trans-Earth insertion from the back side of the moon. The measurements of the state consisted of angles

Table 1 Cislunar simulation parameters and standard deviations

Parameter	Value used in the filter model	Value used to simulate actual environment
rms error in μ_E	0	1.24×10^7 miles^3/hr^2
rms error in μ_M	0	1.52×10^5 miles^3/hr^2
rms error in Earth axes	0.2 miles	0.2 miles
rms error in moon radius	0.2 miles	0.5 miles
rms Earth horizon error	1.2 miles	6.2 miles
rms moon horizon error	0.9 miles	1.9 miles
Sextant rms error	5.0×10^{-5} rad	7.4×10^{-5} rad
Sextant bias	0	4.16×10^{-5} rad

Standard deviation of angular momentum pseudo-measurement = 25 miles^2/hr
Standard deviation of energy pseudo-measurement = 9.439×10^4 (miles/hr)^2
Initial (diagonal) position standard deviations = 5.68 miles
Initial (diagonal) velocity standard deviations = 20.45 miles/hr

Received February 18, 1972; revision received March 6, 1972. This research was partially supported by Manned Spacecraft Center contract NAS 9-4065 T. O. 41.

Index categories: Navigation, Control, and Guidance Theory; Spacecraft Navigation Guidance, and Flight-Path Control Systems.

* Associate Director, Charles Stark Draper Laboratory. Fellow AIAA.

† Staff Scientist, Charles Stark Draper Laboratory. Associate Member AIAA.

‡ Research performed while Master's degree candiate at MIT, currently at Naval Avionics Facility, Indianapolis, Indiana.

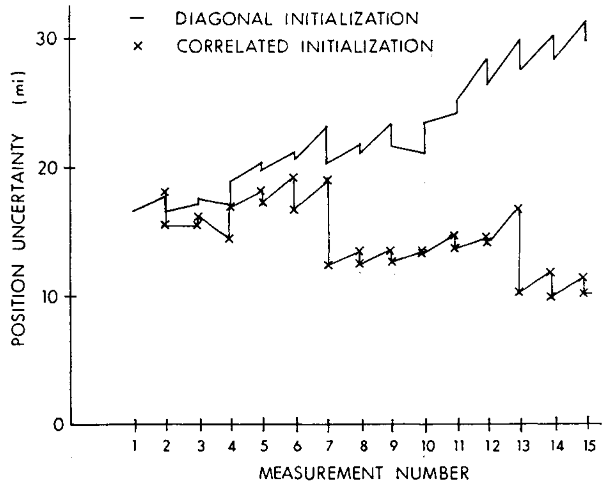

Fig. 1 Comparison of initialization techniques for cislunar position errors.

between various stars of known direction and the horizons of both the Earth and moon. The navigation results using a diagonal initial covariance matrix were compared to those obtained by initializing with a correlated matrix that had been computed by adjoining energy and angular momentum information to the diagonal matrix. The actual position and velocity errors, determined by averaging the results of 25 Monte Carlo computer simulations, are plotted in Figs. 1 and 2 for the first 15 measurements. Included in each sample trajectory are random modeling errors in the gravitational parameters of the Earth and the moon, their radii, their horizons, the sextant rms error and the sextant bias as given in the accompanying table. At the end of this batch of measurements the actual position and velocity errors have been improved by approximately factors of three and two, respectively, demonstrating that the additional information supplied to the diagonal matrix is quite effective in producing a better estimate of the true state.

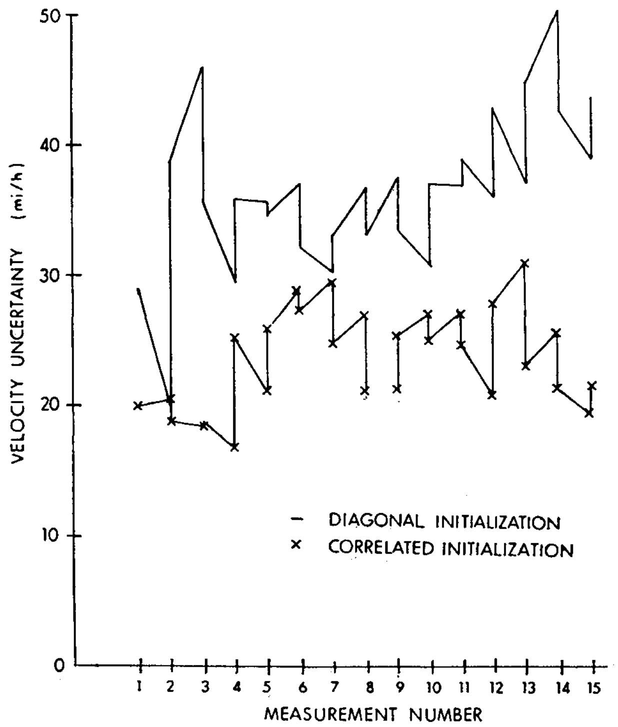

Fig. 2 Comparison of initialization techniques for cislunar velocity errors.

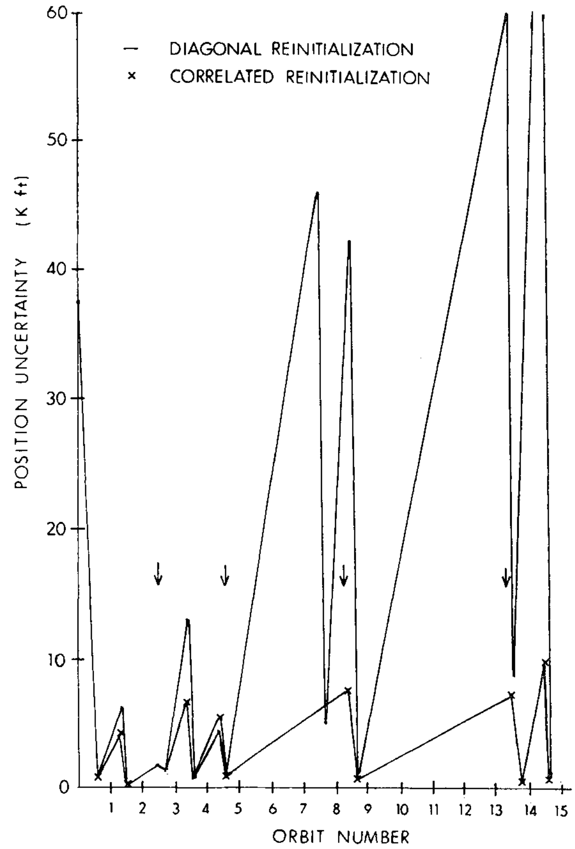

Fig. 3 Comparison of reinitialization techniques for space shuttle orbit navigation position errors.

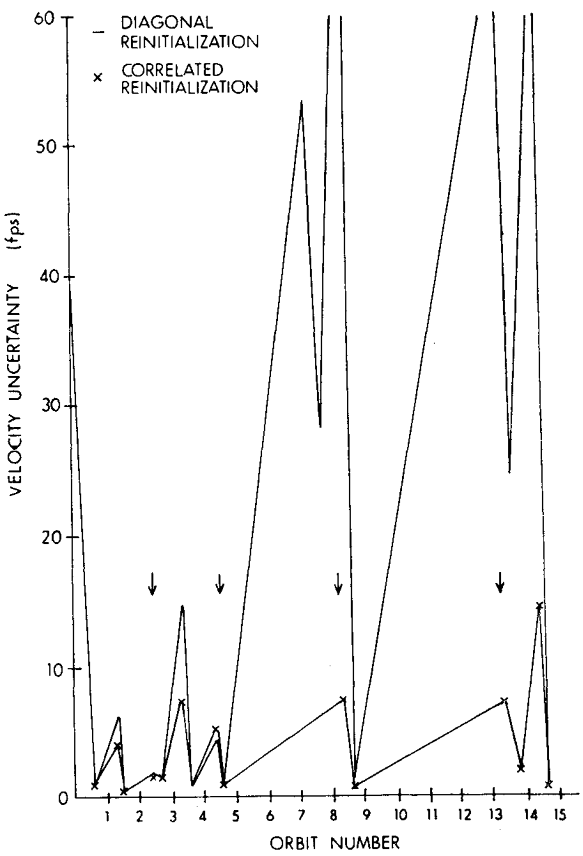

Fig. 4 Comparison of reinitialization techniques for space shuttle orbit navigation velocity errors.

Reinitialization of the error covariance matrix by this method in a space shuttle navigation system, where the measurements consist of range and range-rate to Earth-fixed beacons, was also studied.[3] The position and velocity errors were averaged for 10 Monte Carlo trajectory simulations. Figures 3 and 4 show that the actual errors are improved by an order-of-magnitude in many cases with the benefit of low-error growth rates during regions where measurements are not made. (Reinitializations of the error covariance matrix are indicated by the arrows.)

In summary, the pseudo-measurement technique has been demonstrated to be useful when reinitialization of the error covariance matrix is required to prevent filter divergence. By assuming a diagonal matrix, useful information pertaining to cross-covariance errors is lost. From an engineering standpoint it is desirable to carry as much information as possible through the reinitialization phase. Even though the gains associated with the covariance matrix deteriorate, information pertaining to certain parameters of the problem seem to be inherent in the current best estimate of the state and can be utilized to develop an appropriately correlated matrix with which to restart the navigation problem.

References

[1] Battin, R. H. and Levine, G. M., "Application of Kalman Filtering Techniques to the Apollo Program," *Theory and Applications of Kalman Filtering*, edited by C. T. Leondes, Chap. 14, AGARDograph No. 139, Feb. 1970.

[2] Habbe, J. M., "Kalman Filter Initialization Using Prior Information Applied to Apollo Midcourse Navigation," Thesis-553, May 1971, MIT, Cambridge, Mass.

[3] Muller, E. S. and Philliou, P., Shuttle Orbit Navigation Analysis, Group 23A STS Memo No. 48–71, MIT Charles Stark Draper Lab., Sept. 1971.

Errata

Errata: "An Analysis of Plume-Induced Boundary-Layer Separation"

MICHAEL C. FONG
Lockheed Missiles & Space Company, Sunnyvale, Calif.

[J. Spacecraft Rockets **11**, 1107–1113(1971)]

AN error has been found in Eq. (5) which should read

$$A = M_{ex}{}^{0.5}\left[-0.04\left(\log\frac{P_{ex}}{P_0}\right)^3 + 0.397\left(\log\frac{P_{ex}}{P_0}\right)^2 - 0.579\left(\log\frac{P_{ex}}{P_0}\right)\right] \quad (5)$$

The author regrets that this error was made, but the correct equation was used in obtaining the results presented in the article. Also a typographical error appears in the definition of *Cr* in the Nomenclature section, where the "−" before the parenthesis in the denominator should be a "+."

Received January 26, 1972.